Shopping Round

May 17, 2026

Welcome to the LAMT Shopping Round! In this team event, you are not required to solve every problem. Instead, your team is given a budget to "purchase" problems from our Shopping List.

Exam Rules & Format:

- Your team begins with a starting budget of **$100**. Each problem will have an associated cost in dollars, and a point value, indicated on the exam. Your score will be based on the amount of points your team earns.
- You may choose to buy any combination of problems from the Shopping List, provided the total cost does not exceed your $100 budget. It is impossible to buy every question, so allocate your money carefully.
- The problems are arranged in increasing order of difficulty, with later problems having a higher price. However, they also give significantly more points. The ratio of points to cost is generally increasing.
- After 60 minutes have passed, you must submit the list of problems you wish to buy, along with your final answers for those problems. Only answers to problems which you bought will be scored. For every dollar you spend above $100, 3 points will be subtracted from your score.
- Answers are not necessarily integers, but should be reasonably simplified. If you are unsure whether an answer is in an acceptable form, ask a proctor. There is no penalty for guessing.
- The answer sheet will contain a tiebreaker estimation question. This will only be used to break ties between teams. It does not cost any money, and will not be worth any points.

You have 60 minutes. Good luck, and spend wisely!

Official Team Receipt & Answer Sheet

Team Name: ______________________ **Team ID:** __________

Instructions: Fill in the circle ◯ in the "Buy?" column to purchase a problem. Write answers in the "Final Answer" column. Only answers to purchased problems will be scored. Make sure you **do not** exceed the maximum budget of $100.

Buy?	Q#	Cost	Pts	Final Answer
◯	**1**	$3	4	
◯	**2**	$4	5	
◯	**3**	$5	7	
◯	**4**	$5	8	
◯	**5**	$6	10	
◯	**6**	$6	11	
◯	**7**	$7	13	
◯	**8**	$7	14	
◯	**9**	$8	17	
◯	**10**	$8	18	
◯	**11**	$9	21	
◯	**12**	$9	22	

Buy?	Q#	Cst	Pts	Final Answer
◯	**13**	$10	26	
◯	**14**	$10	27	
◯	**15**	$11	31	
◯	**16**	$11	32	
◯	**17**	$12	36	
◯	**18**	$12	38	
◯	**19**	$13	43	
◯	**20**	$13	45	
◯	**21**	$14	51	
◯	**22**	$14	53	
◯	**23**	$15	59	
◯	**24**	$15	62	

[TIEBREAKER ESTIMATION] Estimate the proportion of teams at LAMT 2026 which decided to buy problem 1 on the Shopping Round, which costed $3, but only had a return of 4 points. Enter your answer as a real number $0.abcdef$, where a, b, c, d, e, and f are digits.

Tiebreaker Answer	

Shopping Round

May 17, 2026

LAMT 2026

1. **[$3, 4]** Find the sum of the digits of

$$67 + 6767 + 676767 + 67676767 + 6767676767.$$

2. **[$4, 5]** Tired of all the 67 brainrot, Albert decides to bring back the early 2010s dank memes (which are obviously better), and writes $9 + 10 = 21$ on a board. However, Vicky, who is a newgen, is confused and reasons that Albert forgot to specify the base of the numbers he wrote. Vicky changes Albert's equation to the factual equation $9_a + 10_a = 21_b$, where a and b are positive integers (so 9 and 10 are read in base a and 21 in base b). Find the smallest possible value of $a + b$.

3. **[$5, 7]** There are some pennies, nickels, dimes, and quarters sitting on a table, with seven coins in total. If Luke takes a set of 5 coins from the table, the largest possible total he can attain is M, and the smallest is m. Given that $M - m = 14$, find the largest possible total value of the seven coins, in cents.

4. **[$5, 8]** Order the following from least to greatest:

$$1)\ 57! \qquad 2)\ 5^{7!} \qquad 3)\ (5^7)! \qquad 4)\ (5!)^7.$$

Enter your answer as a permutation of the digits 1234. For instance, if you think $5^{7!} < 57! < (5^7)! < (5!)^7$ your answer will be 2134.

5. **[$6, 10]** Regular hexagon $LOSANG$ has side length 4. Point E lies on diagonal OA such that $OE = \sqrt{3}$. Find the area of GEN.

6. **[$6, 11]** Find the unique ordered pair (m, n) of positive integers $m < n$ for which m and n both have exactly 16 positive divisors, and $m + n = 856$.

7. **[$7, 13]** Patrick puts each integer from 1 through 9 in a distinct cell of a 3×3 grid. He then starts in the cell labeled 1, and every second he travels to the neighboring cell with the largest value. Suppose his path is $1 \to 3 \to 5 \to 7 \to 9$. Find the number of ways Patrick could have filled in the grid.

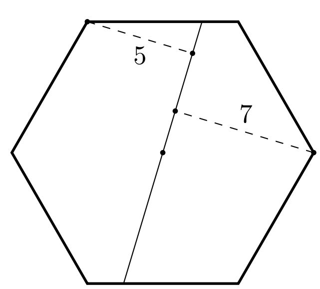
8. **[$7, 14]** An island is located at each point (x, y) for which x and y are positive integers satisfying $x + y \leq 7$. Pierre builds two types of one way bridges between islands:

- from each point $(x + 1, y)$, a one way bridge may be built to (x, y), or
- from each point $(x + 1, y - 1)$, a one way bridge may be built to (x, y).

Find the number of bridge arrangements satisfying the following: for every island in S, there is exactly one directed path from $(6, 1)$ to that island that does not reuse any bridge.

9. **[$8, 17]** Let $s_b(n)$ denote the sum of the digits of n when written in base b. Find the smallest positive integer n for which $\frac{s_2(n)}{s_4(n)} = \frac{5}{9}$. Express your answer in base 10.

10. **[$8, 18]** Anthony draws a regular hexagon $ABCDEF$ and makes a cut of length 13 through the center. The heights from A and C to the cut are in the ratio $5 : 7$. Find the side length of the hexagon.



11. **[$9, 21]** Find the remainder when $\frac{669!}{336! \cdot 334!}$ is divided by 67.

12. **[$9, 22]** Suppose a, b, c are positive integers satisfying $\gcd(a, b) = 6$, $\gcd(a, c) = 15$, and $\operatorname{lcm}(b, c) = 180$. Find the minimum possible value of $a + b + c$.

13. **[$10, 26]** Suppose x is a real number. Find the sum of all possible values of x^2, given that

$$\frac{x^3 + 2}{5} = \sqrt[3]{5x - 2}.$$

14. **[$10, 27]** Aryan arranges 20^3 unit cubes into a large $20 \times 20 \times 20$ cube. Find the number of ordered pairs (A, B) of two distinct unit cubes for which every contiguous $4 \times 4 \times 4$ of unit cubes that contains A also contains B.

15. **[$11, 31]** Five athletes are ranked based on their speed and strength (where 1 is the best, and 5 is the worst). They all know each other's ranks, and make the following truthful statements:

- Amara: I have the same rank in both categories.
- Billy: I'm first in something.
- Cris: Dante is the only person stronger and faster than me.
- Dante: I'm not first in anything.
- Elena: I'm the weakest (in strength).

Let A, B, C, D, and E be the product of the speed rank and strength rank for Amara, Billy, Cris, Dante, and Elena, respectively. Find $A + 2B + 3C + 4D + 5E$.

16. **[$11, 32]** Let ABC be an isosceles triangle with $\angle B = 90°$. Point P lies inside ABC with $\angle BPC = 135°$ and $AP = AB = 1$. Find BP.

17. **[$12, 36]** Find

$$\sum_{n=1}^{30} \binom{n}{\lceil \frac{n+1}{2} \rceil} = \binom{1}{1} + \binom{2}{2} + \binom{3}{2} + \cdots + \binom{28}{15} + \binom{29}{15} + \binom{30}{16}.$$

18. **[$12, 38]** Find the sum of all positive integers $n < 210$ for which $\gcd(n^n - 1, 210) = 5$.

19. **[$13, 43]** Let a, b, and c be real numbers satisfying

$$a + ab = 4, \qquad 2b + bc = 5, \qquad 3c + ca = 6.$$

Find the sum of all possible values of $(abc)^2$.

20. **[$13, 45]** Find the sum of all positive integers $n < 455$ for which the remainders when n is divided by 5, 7, and 13 all sum to 11.

21. **[$14, 51]** Let a, b, and c be positive real numbers such that

$$a^2 + b^2 - ab = 49, \qquad b^2 + c^2 + bc\sqrt{3} = 225, \qquad a^2 + c^2 + ac\sqrt{3} = 400.$$

Find the value of $a\sqrt{900 - b^2} + b\sqrt{1600 - a^2}$.

22. **[$14, 53]** There are mirrors on sides AB and CD of rectangle $ABCD$ facing the interior, with $AD = 16$ and $AB = 25$. Seven points split AD into 8 segments of length 2, and a laser is shot from each point. Each laser forms a $45°$ angle with AD and goes towards AB or CD, each with probability $\frac{1}{2}$. Find the the expected number of times two lasers intersect before hitting side BC.

23. **[$15, 59]** Let $ABCDEF$ be a hexagon with area 100 and opposite sides parallel. Suppose $AB = 2DE$, $CD = 2FA$, $EF = 2BC$. The circumcircles of ABC and DEF intersect at points P and Q. Suppose $PQ \parallel FA$, and line PQ splits $ABCDEF$ into two polygons. Find the area of the smaller polygon.

24. **[$15, 62]** Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. Find the remainder when

$$\left\lfloor \left(15 + 6\sqrt{6} + 10\sqrt{2} + 8\sqrt{3}\right)^{1919} + \left(15 + 6\sqrt{6} - 10\sqrt{2} - 8\sqrt{3}\right)^{1919} \right\rfloor$$

is divided by 100.